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present

Maple as a **Computer Algebra System**

and

Pen as an **Input Device**

* * *

starring in

Pen-based Rearrangement of 2D Math Expressions in Maple

(preview)

Epigraph:

... Breaking rules can serve for the purpose of good as well as … research.

Idea

Final goal is to be able to handle with sufficient ease two-dimensional mathematical expressions from a pen-based input.

Scene A

For example having an equation like

$$A + F(x) - \sin(x^2) = B \; ,$$

one may drag one term to the other side of the equation sign, so it should appear on the other side with an opposite sign:

$$F(x) - \sin(x^2) = B - A$$

Scene B

or having a formula like

$$\frac{x(e^{2x}-\sin^4 x)^3}{e^x+\sin^2 x}$$

one may want to simplify it by circling the power to the second term in the numerator and crossing the whole denominator:

$$\frac{x(e^{2x}-\sin^4 x)^3}{\cancel{e^x+\sin^2 x}}$$

so the resulting formula will look like

$$x(e^{2x}-\sin^4 x)^2(e^x-\sin^2 x)$$

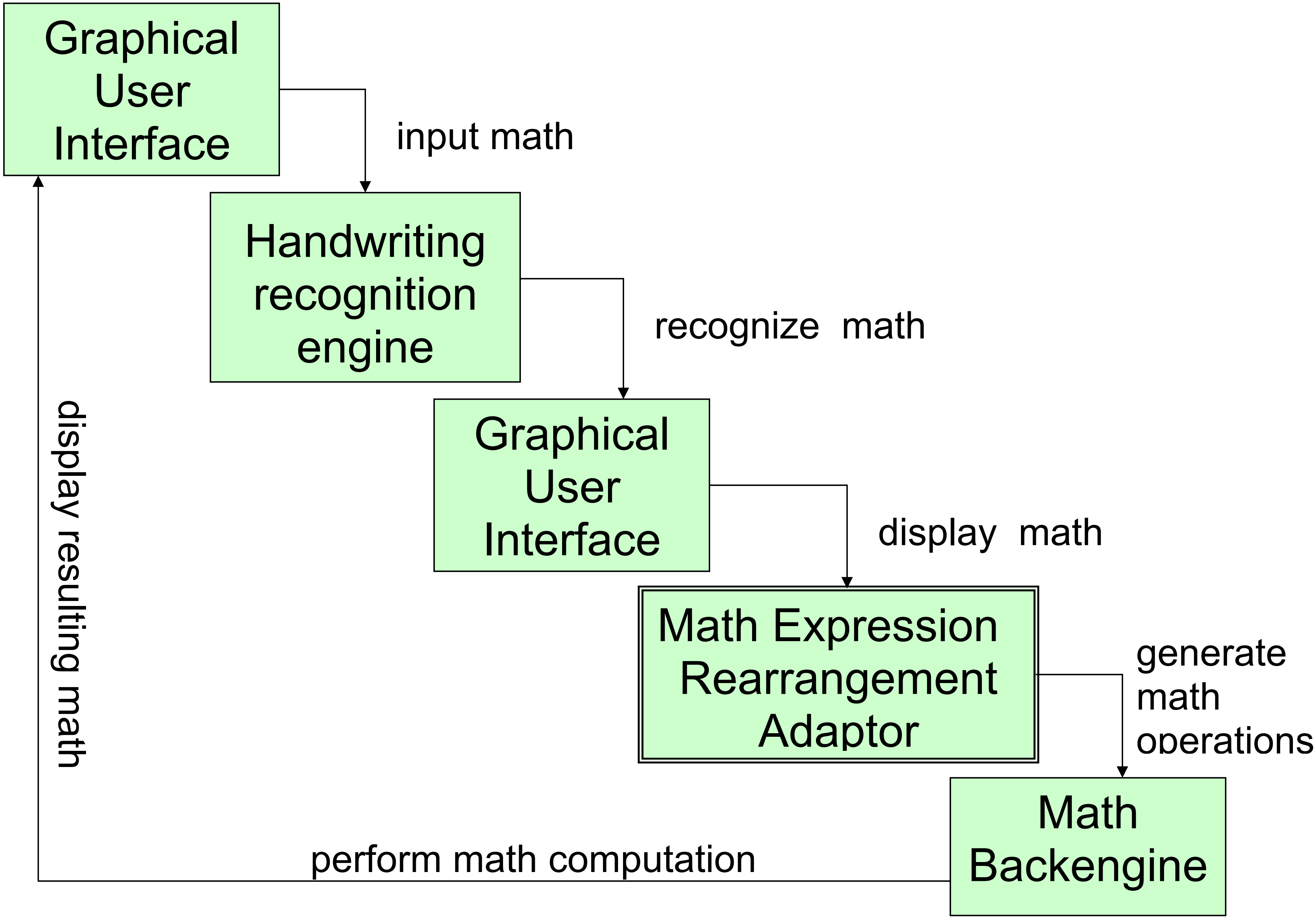
Plot

To achieve this goal we have to provide **software tools for 2D math expression rearrangement.** These tools can to be integrated into a pen-based math system.

Main Characters

- *Graphical User Interface* to capture math input, show math output and handle pen gestures on displayed math.
- *Math handwriting recognition engine* to recognize math input and build math expression from it.
- *Math backengine* to perform necessary background computation involved in expression rearrangement

Scenario


Graphical User Interface
input math
Handwriting recognition engine
recognize math
Graphical User Interface
display math
Math Expression Rearrangement Adaptor
generate math operations
Math Backengine
perform math computation
display resulting math

Intrigue

Problem:

Math handwriting recognition tools are currently under development, but we need to have a source of <u>meaningful</u> math editing problems to serve as a realistic base expressions to be rearranged.

Solution:

Use math that has been already parsed and rendered in math environment.

An experiment:

As a base for math expression rearrangement we use 2D output from a Maple computation (which appears in blue color on Maple worksheet)

Prologue

To be able rearrange math expression we need the information about its math content as well as its math presentation. It is especially important for ambiguous cases: e.g. x^2 (power vs. upper index), and $f\,x$ ($f\,(x)$ vs. $f{\cdot}x$ vs. name $f\,x$)

Every output math expression in Maple has two tree structures associated with it:

- *Maple math model* stores presentation information about math. It serves to render math in GUI, and is based on the W3 consortium MathML standard.
- *DAG* (**d**irect **a**cyclic **g**raph) encodes math content.

For example an expression $\sin(x^2) - 1 = y$ appears as

1. Math Model

```
MROW
 ├─MI [sin]
 │ MROW
 │  ├─MO [(]
 │  ├─MROW
 │  │  └─MSUP
 │  │     ├─MI [x]
 │  │     └─MN [2]
 │  └─MO [)]
 ├─MO [-]
 ├─MN [1]
 ├─MO [=]
 └─MI [y]
```

2. DAG

```
EQUATION
 ├─ SUM
 │   ├─ FUNCTION
 │   │   ├─ NAME: sin
 │   │   └─ EXPSEQ
 │   │       └─ PROD
 │   │           ├─ POWER
 │   │           │   ├─ NAME: x
 │   │           │   └─ INTPOS: 2
 │   │           └─ INTPOS: 1
 │   ├─ INTPOS: 1
 │   ├─ INTNEG: -1
 │   └─ INTPOS: 1
 └─ NAME: y
```

Script

Once the user has selected a subexpression in Maple output, s/he then can choose to perform one of the following actions:

- get an "operand" path to the selected subexpression in root formula:

ex: $\sin(x^2 - y^2) - \sin(x^2)\cos(y^2)$ → **`op([1,1,2,2,2],%)`**

- replace the selection*

ex: $\sin(x^2 - y^2) - \sin(x^2)\cos(y^2)$

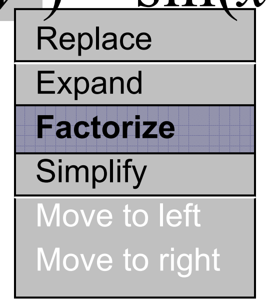
*In this case the track of changes to Maple output will be noted to the Maple input: it will be changed to

```
> sin(x^2-y^2)-sin(x^2)*cos(y^2):
  subsop([2,2]=<replacing expr>,%);
```

- Apply *value preserving transformations* to the selected subexpression

  I.e. *expand* / *factorize* / *simplify* the selection and replace it with the result of a chosen operation **

  ex: $$\sin(x^2 - y^2) - \sin(x^2)\cos(y^2)$$



Clicking on the highlighted operation will produce a new expression in Maple output

$$\sin((x - y)(x + y)) - \sin(x^2)\cos(y^2)$$

** and also add new directive to corresponding Maple input if it exists:

```
> sin(x^2-y^2)-sin(x^2)*cos(y^2):
  subsop([1,1]=factor(op([1,1],%)),%);
```

- Dragging the selection in other side of the equation/inequality in case of sum as main operator:

```
>Int(sin(x^2),x)-x=int(sin(x^2),x)+x^2;
```

$$\int \sin(x^2)dx - x = \frac{1}{2}\sqrt{2\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\ \mathrm{x}}{\sqrt{\pi}}\right) + x^2$$

will produce

```
>Int(sin(x^2),x)-x=int(sin(x^2),x)+x^2:
 lhs(%)-op(1,rhs(%)=rhs(%)-op(2,lhs(%));
```

$$\int \sin(x^2)dx - \frac{1}{2}\sqrt{2\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\ \mathrm{x}}{\sqrt{\pi}}\right) = x^2 - x$$

- Dragging the selection in other side of the equation in case of product as main operator

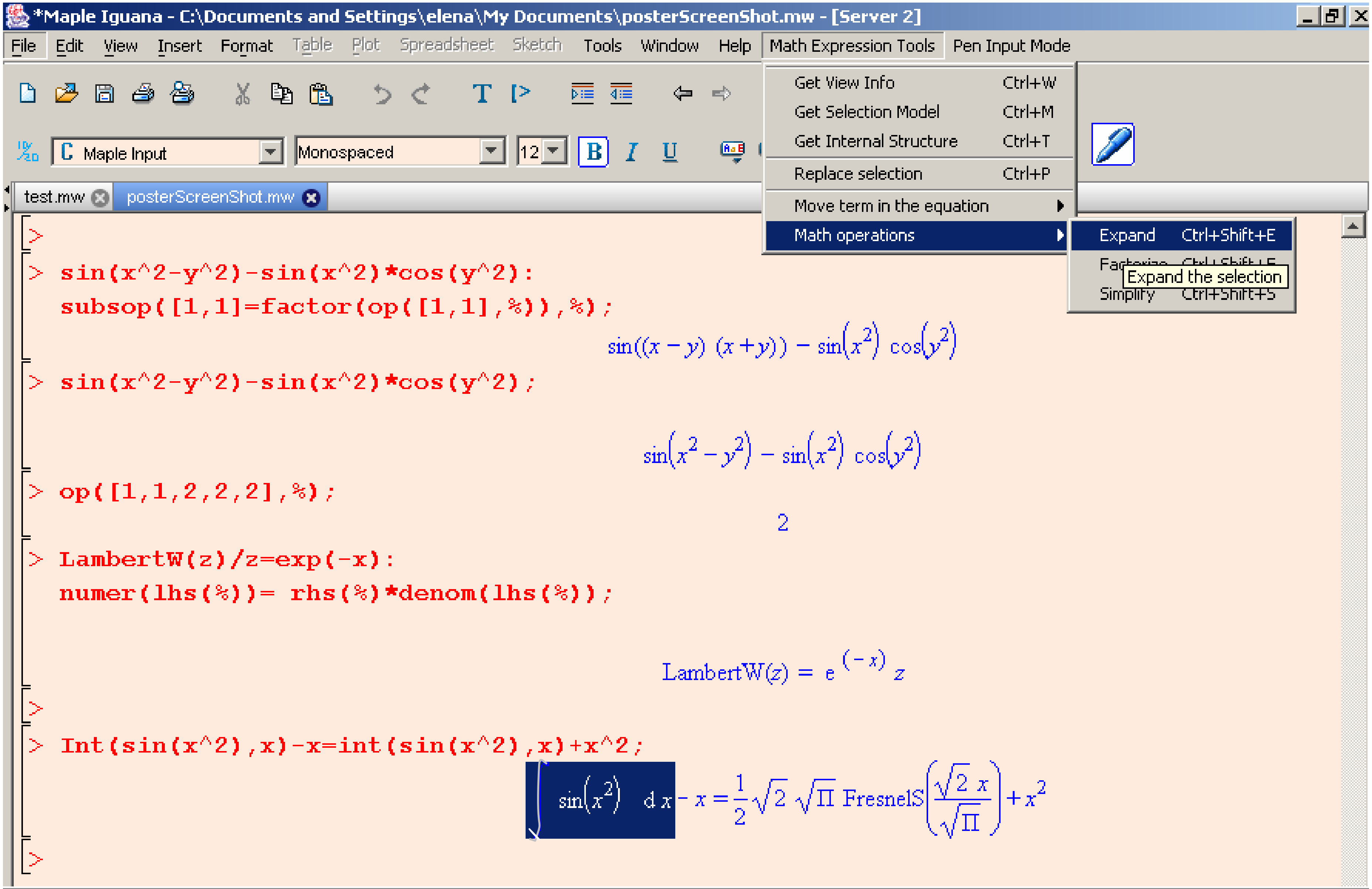
$$\frac{LambertW(z)}{z} = e^{-x}$$

will give

```
> LambertW(z)/z=exp(-x):
  numer(lhs(%))= rhs(%)*denom(lhs(%));
```

$$LambertW(z) = e^{-x} z$$

Action!


*Maple Iguana - C:\Documents and Settings\elena\My Documents\posterScreenShot.mw - [Server 2]
File Edit View Insert Format Table Plot Spreadsheet Sketch Tools Window Help Math Expression Tools Pen Input Mode
Get View Info Ctrl+W
Get Selection Model Ctrl+M
Get Internal Structure Ctrl+T
Replace selection Ctrl+P
Move term in the equation
Math operations
Expand Ctrl+Shift+E
Expand the selection
Simplify Ctrl+Shift+S
Maple Input
Monospaced
12
test.mw
posterScreenShot.mw
> sin(x^2-y^2)-sin(x^2)*cos(y^2):
subsop([1,1]=factor(op([1,1],%)),%);
$\sin((x-y)\,(x+y)) - \sin(x^2)\,\cos(y^2)$
> sin(x^2-y^2)-sin(x^2)*cos(y^2);
$\sin(x^2-y^2) - \sin(x^2)\,\cos(y^2)$
> op([1,1,2,2,2],%);
2
> LambertW(z)/z=exp(-x):
numer(lhs(%))= rhs(%)*denom(lhs(%));
$\mathrm{LambertW}(z) = e^{(-x)}\,z$
> Int(sin(x^2),x)-x=int(sin(x^2),x)+x^2;
$\int \sin(x^2)\,dx - x = \frac{1}{2}\sqrt{2}\,\sqrt{\Pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\Pi}}\right) + x^2$

Behind the Stage

To integrate tools for math expression rearranging with Maple we needed to

- create three Java packages for adding new controllers, components and event handlers to the Maple worksheet GUI.
- provide entry points in existing Maple GUI to plug and activate new tools *without performing major changes to the original Maple Iguana code*
- develop a middleware to generate Maple instructions generated from pen gestures and menu calls to be executed in Maple kernel.
- hook-up Maple kernel and a new GUI tools created.

In next series…

Further work in this direction:

- Allow Maple worksheet to capture ink strokes from Pen input device to determinate selections in math expression (responsible: Elena Smirnova)
- Be able to parse and disambiguate 2D math input

  (Maplesoft Inc. and Co.)
- Develop real math handwriting recognition tools (ORCCA Pen-Math project and al.)

To be continued…